IFT-07-02
UCRHEP-T434

Note on the strong CP problem from a 5-dimensional perspective
– the gauge-axion unification –

Bohdan GRZADKOWSKI*
Institute of Theoretical Physics, University of Warsaw, Hoża 69, PL-00-681 Warsaw, Poland

José WUDKA†
Department of Physics, University of California, Riverside CA 92521-0413, USA

We consider 5 dimensional gauge theories where the 5th direction is compactified on an interval. The Chern-Simons (CS) terms (favored by the naive dimensional analysis) are discussed. A simple scenario with an extra $U(1)_X$ gauge field that couples to $SU(3)_{\rm color}$ through a CS term in the bulk is constructed. The extra component of the Abelian gauge field plays a role of the axion (gauge-axion unification), which in the standard manner solves the strong CP problem easily avoiding most of experimental constraints. Possibility of discovering the gauge-unification at the LCH is discussed.

PACS numbers: 11.10.Kk, 11.15.-q, 12.10.-g, 14.80.Mz

I. INTRODUCTION

In the Standard Model (SM), the Higgs mechanism is responsible for generating fermion and vector-boson masses. Although the model is renormalizable and unitary, it has severe naturality problems associated with the so-called "hierarchy problem". At loop-level this problem reduces to the fact that the quadratic corrections tend to increase the Higgs boson mass up to the UV cutoff of the theory. Extra dimensional extensions of the SM offer a novel approach to gauge symmetry breaking in which the hierarchy problem could be either solved or at least reformulated in terms of the geometry of the higher-dimensional space.

Other inherent problems of the SM could also be addressed in extra-dimensional scenarios. For instance, within the SM the amount of CP violation is not sufficient to explain the observed baryon asymmetry [1], the gauge-Higgs unification scenario offers a possible solution since in such models the geometry can be a new source of explicit and spontaneous CP violation [2]. In this note we shall prove that the strong CP problem could be solved introducing an appropriate Chern-Simons (CS) terms in 5D [1]. The scenario leads to an attractive possibility of gauge-axion unification.

II. HIERARCHY OF EFFECTIVE OPERATORS

We will first consider models in $D = 5$ dimensions with fermions, gauge bosons and scalars propagating throughout the D-dimensional bulk, and some unspecified matter localized on lower dimensional manifolds (branes). Though these models are non-renormalizable it is possible to define a hierarchy of possible terms in the Lagrangian that allows for a proper perturbative expansion; the procedure is a simple application of the arguments used in the naive dimensional analysis (NDA) [4], see the Appendix. This hierarchy is specified by assigning to each gauge invariant operator an index $s = d_c + b' + (3f/2) - 4$, ($d_c$ is the number of covariant derivatives, f and b' the number of fermion and scalar fields). As it is shown in the Appendix the least suppressed operators are those that have the index $s = 0$:

$$F^2; \quad \bar{\psi} D\psi; \quad |D\phi|^2; \quad \bar{\psi}\phi\psi; \quad \phi^4 \,, \tag{1}$$

where F denotes the generic gauge tensor, ϕ a generic scalar, and ψ generic fermions.

The $s = 1$ operators not containing scalar fields are (A denotes a generic gauge field)

$$AF^2; \quad \bar{\psi} F\psi \,, \tag{2}$$

*Electronic address: bohdan.grzadkowski@fuw.edu.pl
†Electronic address: jose.wudka@ucr.edu
[1] For other attempts to solve the strong CP problem by 5 dimensions see [3].

whose coefficients are naturally suppressed by $1/(24\pi^3)$, together with all brane terms, presumably including the SM Lagrangian multiplied by $l_4^{-1}\delta(y-y_o)$. The first operator in (2) corresponds to the 5-dimensional Chern-Simons (CS) term, while the second includes all magnetic-type couplings. Operators of index $s=1$ containing ϕ are of the form $D^4\phi$, $D^2\phi^3$, or $D\bar{\psi}\psi\phi$.

The NDA argument favors the presence of a CS term (if only 5D vector bosons are present the CS term is the only bulk operator with index $s=1$), with a coefficient as large as $1/(24\pi^3)$. Of course, it is still possible that there exist additional symmetries that forbid this term, however if present, the CS term can generate interesting effects.

Hereafter we shall consider a 5D model containing $U(1)_X$ and $SU(3)_{\rm color}$ bulk gauge fields, denoted by X and G respectively. Application of the NDA for this case (where there are no bulk fermions) yields the following action up to index $s=1$

$$\begin{aligned} S \;=\; & \int_{X^5} d^5x \left\{ -\frac{1}{4} X_{MN}X^{MN} - \frac{1}{2}\mathrm{Tr}\left[G_{MN}G^{MN}\right] + \right. \\ & - \frac{1}{24\pi^3}\epsilon^{LMNPQ}\left[c_1 g_5^{'} g_5^2 X_L \mathrm{Tr}\left(G_{MN}G_{PQ}\right) + c_2 g_5^{'3} X_L X_{MN} X_{PQ} + \right. \\ & \left.\left. + \; c_3 g_5^3 \mathrm{Tr}\left(G_L G_{MN} G_{PQ} + \frac{i}{2} G_L G_M G_N G_{PQ} - \frac{1}{10} G_L G_M G_N G_P G_Q\right)\right]\right\} + \frac{1}{16\pi^2} S_{\rm brane} \end{aligned} \tag{3}$$

where X_{MN} and G_{MN} are, respectively, the field strength tensors for the Abelian and non-Abelian groups [2] with the 5D gauge couplings respectively denoted by $g_5^{'}$ and g_5; $c_{1,2,3}$ are undetermined numerical constants, presumably of $O(1)$. In our specific applications we will consider models constructed on the space-time $X^5 = M^4 \times [0,R]$, and we will concentrate on the "mixed" Chern-Simons term proportional to $g_5^{'} g_5^2$. We will assume that all SM fields are neutral under $U(1)_X$. Hereafter, whenever possible, in order to make the analysis as model independent as possible, we will avoid referring to any details of the embedding of the SM into 5D. The only assumption we make is that the SM is localized on one or perhaps both ends of the interval $[0,R]$.

III. SOLVING THE STRONG CP PROBLEM FROM A 5D PERSPECTIVE

As shown above, the NDA favors the CS term as an operator of index $s=1$. We will argue that the presence of this term allows for a simple solution to the strong CP problem.

As it is well known, in a basis where the Yukawa matrices are diagonal, the phases of the Kobayashi-Maskawa matrix are responsible for all electroweak CP violation effects. There is, however, an additional ("strong") CP-violating term allowed by the symmetries of the 4D SM Lagrangian:

$$\mathcal{L}_{\rm QCD\ CP} = \theta \frac{\alpha_s}{16\pi} \mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right), \tag{4}$$

where $G_{\mu\nu}$ is the QCD field strength tensor, $\tilde{G}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}/2$, and $\alpha_s \equiv g^2/(4\pi)$ for g the SM 4D QCD gauge coupling constant. In the process of diagonalizing the Yukawa matrices, quark fields undergo a chiral rotation, which generates the same structure as in (4) (within the path-integral formulation this results from a non-trivial Jacobian for the fermionic measure [5]); therefore the total effect of the strong CP violation is parameterized by the effective coefficient $\theta_{\rm eff} \equiv \theta + \theta_{\rm weak}$. The experimental data (EDM of the neutron) indicates that $|\theta_{\rm eff}| \lesssim 10^{-9}$ [6]; this is referred to as the strong-CP "problem" since none of the symmetries of the SM requires such a strong suppression.

Models in extra dimensions offer new possibilities to solve this problem due to a possibility of constructing the Chern-Simons terms. Specifically, we will assume that the color gauge fields G_N^a propagate in the bulk, but that the rest of the SM fields are confined to one or two branes located at $y=0$ and $y=R$. In addition we assume the presence of an Abelian gauge field X_N also propagating in the bulk. For the 5D models being considered here, the QCD strong-CP term (4) can be written as follows:

$$S_{\rm brane} = \frac{\alpha_s}{16\pi^2} \int d^5x \left[\theta_L \delta(y) + \theta_R \delta(y-R)\right] \mathrm{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right), \tag{5}$$

where $\theta_{R,L}$ are constant parameters.

---

[2] The convention for the antisymmetric tensors which we follow is such that $\epsilon_{01234} = \epsilon_{0123} = 1$ for the metric tensor $\eta_{MN} = \mathrm{diag}(1,-1,-1,-1,-1)$ and $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. We assume that the non-Abelian group generators, T^a are Hermitian and normalized according to $\mathrm{Tr}T^aT^b = 2^{-1}\delta_{ab}$.

Among the various terms in (3) we will concentrate on the effects of the mixed CS term:

$$S_{\rm CS} = -\frac{g_5^{'} g_5^2 c_1}{24\pi^3} \int_{X^5} d^4x \, dy \, \epsilon^{LMNPQ} X_L {\rm Tr}\left(G_{MN} G_{PQ}\right) . \tag{6}$$

The action (6) is not automatically gauge invariant under the $U(1)_X$. However, using the Bianchi identity $\epsilon^{NMQPR} D_Q G_{PR} = 0$, one can show that under the Abelian transformation

$$X_L \to X_L' = X_L + \partial_L \lambda_X \tag{7}$$

the change in $S_{\rm CS}$ is localized on the boundary of the space [3].

$$\delta S_{\rm CS} = \frac{g_5^{'} g_5^2 c_1}{24\pi^3} \int_{M^4} d^4x \, \lambda_X \, \epsilon^{\mu\nu\alpha\beta} {\rm Tr}\left(G_{\mu\nu} G_{\alpha\beta}\right) \Big|_{y=0}^{y=R} \tag{8}$$

There are various ways of insuring that this vanishes. One can, for example, add an appropriate set of chiral fermions on the two branes; in this case the anomaly generated by these fermions can be adjusted so that it cancels (8), see e.g. [7]. Brane scalars can be also arranged to have the same effect [3], [7] provided they couple to $\epsilon^{\mu\nu\alpha\beta}{\rm Tr}(G_{\mu\nu}G_{\alpha\beta})$. A simpler alternative, which we will adopt here, is to impose appropriate boundary conditions such as $\lambda_X {\rm Tr}(G^2)|_{y=0} = \lambda_X {\rm Tr}(G^2)|_{y=L}$.

Variation of the total action (3) with $c_2 = c_3 = 0$ and $c_1 = 1$ leads to the following equations of motion for the gauge fields:

$$D_B G^{BA} = J^A + \text{brane terms} \qquad \text{and} \qquad \partial_B X^{BA} = j^A + \text{brane terms}\,, \tag{9}$$

with the following Chern-Simons currents

$$J^A = \frac{g_5^{'} g_5^2}{24\pi^3} \epsilon^{ABCDE} X_{BC} G_{DE}; \qquad j^A = \frac{g_5^{'} g_5^2}{24\pi^3} \epsilon^{ABCDE} {\rm Tr}\left(G_{BC} G_{DE}\right) \tag{10}$$

The brane terms in (9) originate from possible couplings of the bulk gauge fields to the fields localized on the branes.

For the extremum of the action the following boundary conditions (BC) must be fulfilled:

$$\mathrm{tr}\left[\left(G_{4\mu} - \frac{g_5^{'} g_5^2}{6\pi^3} X^\nu \tilde{G}_{\mu\nu}\right)\delta G^\mu\right]\Bigg|_{y=0}^{y=R} = 0 \qquad \text{and} \qquad X^{4\mu}\delta X_\mu\big|_{y=0}^{y=R} = 0 \tag{11}$$

Here we will restrict ourselves to theories containing massless zero-modes (gluons) of the non-Abelian gauge field. This implies a unique choice of BC for $SU(3)_{\rm color}$:

$$\partial_y G^a_\mu|_{y=0,R} = 0, \qquad G^a_4|_{y=0,R} = 0.\,; \tag{12}$$

these conditions imply $G^a_{4\mu}|_{y=0,R} = 0$. For the Abelian field we require

$$X_\mu|_{y=0,R} = 0, \qquad \partial_y X_4|_{y=0,R} = 0\,, \tag{13}$$

so that $X_{\mu\nu}|_{y=0,R} = 0$. It follows that the BC (11) are satisfied [4].

The resulting Kaluza-Klein (KK) expansions read

$$\begin{array}{ll} G^a_\mu(x,y) = R^{-1/2} \sum_{n=0} d_n G^{a\ (n)}_\mu(x) \cos m_n y & G^a_4(x,y) = R^{-1/2}\sqrt{2} \sum_{n=1} G^{a\ (n)}_4(x) \sin m_n y \\ X_\mu(x,y) = R^{-1/2}\sqrt{2} \sum_{n=1} X^{(n)}_\mu(x) \sin m_n y & X_4(x,y) = R^{-1/2} \sum_{n=0} d_n X^{(n)}_4(x) \cos m_n y \end{array} \tag{14}$$

where $m_n = \pi n/R$ and $d_n = 2^{(1-\delta_{n,0})/2}$. The zero-mode $G^{a\ (0)}_\mu(x)$ is the standard 4D gluon; it is also clear that the model also contains a massless 4D scalar $X^{(0)}_4(x)$.

---

[3] This assumes that λ_X is not a constant.
[4] We thank Kin-ya Oda for a discussion at this point.

Let's focus now on the Abelian gauge transformations. In order to preserve the BC, the gauge function $\lambda_X(x,y)$ must satisfy the following constraints:

$$\partial_\mu \lambda_X|_{y=0,R} = 0, \qquad \partial_y^2 \lambda_X|_{y=0,R} = 0 \tag{15}$$

That implies a corresponding KK expansion for the Abelian gauge function

$$\lambda_X(x,y) = \sum_{n=1} \lambda_X^{(n)}(x) \sin m_n y + \beta y \tag{16}$$

where β is a constant. The 4D vector and scalar fields transform as

$$X_\mu^{(n)} \to X_\mu^{(n)} + \frac{1}{\sqrt{2}} \partial_\mu \lambda_X^{(n)} \qquad X_4^{(n)} \to \begin{cases} X_4^{(0)} + \beta & \text{for} \quad n = 0 \\ X_4^{(n)} + \frac{m_n}{\sqrt{2}} \lambda_X^{(n)} & \text{for} \quad n > 0 \end{cases} . \tag{17}$$

In the following we will take $\beta = 0$, which is the simplest condition ensuring the gauge symmetry of the CS action [5].

In order to discuss phenomenological predictions of the model let us expand the CS action into KK modes:

$$S_{\rm CS} = \frac{R}{12\pi^3} \frac{g_5^{'}}{R^{1/2}} \frac{g_5^2}{R} c_1 \int d^4x \left[X_4^{(0)}\, {\rm Tr} G_{\mu\nu}^{(0)} \tilde{G}^{\mu\nu\,(0)} + 2\partial_\mu X_4^{(0)} \sum_{n=1}^{\infty} {\rm Tr} G_\nu^{(n)} D_\rho G_\sigma^{(n)} \epsilon^{\mu\nu\rho\sigma} - 4 {\rm Tr} \tilde{G}^{\mu\nu\,(0)} \sum_{n=1}^{\infty} \Theta_{\mu\nu}^{(n)} + \cdots \right] \tag{18}$$

where

$$D_\mu \equiv \partial_\mu + ig \left[G_\mu^{(0)}, \cdot \right] \qquad G_{\mu\nu}^{(0)} \equiv \partial_\mu G_\nu^{(0)} - \partial_\nu G_\mu^{(0)} + ig \left[G_\mu^{(0)}, G_\nu^{(0)} \right] \tag{19}$$

for $g = g_5/\sqrt{R}$ and

$$\Theta_{\mu\nu}^{(n)} \equiv \frac{1}{2} \left[\left(\partial_\mu X_4^{(n)} G_\nu^{(n)} - \partial_\nu X_4^{(n)} G_\mu^{(n)} \right) - \left(\partial_\mu X_\nu^{(n)} G_4^{(n)} - \partial_\nu X_\mu^{(n)} G_4^{(n)} \right) - m_n \left(X_\mu^{(n)} G_\nu^{(n)} - X_\nu^{(n)} G_\mu^{(n)} \right) \right] \tag{20}$$

Ellipsis in (18) stands for terms (irrelevant for any practical applications) that involve four non-zero KK modes. Expanding the kinetic terms of (3), one can verify that indeed $G_{\mu\nu}^{(0)}$ corresponds to the SM QCD gluon (which is present due to our having adopted (12)), while $X_4^{(0)}(x) = a(x)$ can play the role of the axion. The lowest-order terms conform the usual QCD action, the axion kinetic term and the axion-gluon interactions [6]:

$$S_{\rm low}^{(0)} = \int_{M^4} \left\{ -\frac{1}{2} {\rm Tr} \left(G_{\mu\nu} G^{\mu\nu} \right) + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{\alpha_s}{16\pi} \left(\frac{a}{f_a} + \theta_{\rm eff} \right) {\rm Tr} \left(G_{\mu\nu} \tilde{G}^{\,\mu\nu} \right) \right\} , \tag{21}$$

where $\theta_{\rm eff} \equiv \theta_L + \theta_R$ and we dropped the (0) superscript in G. Adopting the NDA estimation of the CS coefficient one obtains for the axion decay constant

$$f_a^{-1} = \frac{16 g^{'}}{3\pi} R \tag{22}$$

where $g^{'}$ is the 4D Abelian gauge coupling, $g' = g_5'/\sqrt{R}$, and $\alpha_s = g^2/(4\pi)$. Note that for this mechanism of axion generation to work, the extra Abelian gauge symmetry must be broken by the boundary conditions (Scherk–Schwarz breaking) so that no additional massless vector boson associated with X_μ is present. The only low-energy remnant of X_M is the axion $a(x)$. The crucial advantage of the model presented here is the unification of the axion and the $U(1)$ 5D gauge field. There are serious attempts to construct in 5D a realistic gauge-unification theory [8]. Those models combined with the scenario discussed here could provide an interesting alternative for a theory of electroweak interactions that offers the scalar sector of 4D theory fully unified with a gauge fields (solving the hierarchy problem [8]

---

[5] This is also a natural choice for S^1/Z_2 orbifold models since it insures that $X_\mu(x,-y) = -X_\mu(x,y)$, $X_4(x,-y) = X_4(x,y)$ and $X_N(x, y+2R) = X_N(x,y)$ are preserved under gauge transformations.

[6] It turns out that each term in the KK expansion of (5) is a total derivatives (as they emerge form the full derivative ${\rm Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]$). Only the zero-mode contribution will be relevant as it contributes to the effective non-perturbative axion potential, other terms could be dropped.

and the strong CP problem at the same time). As it will be discussed below the gauge-axion unification is consistent with the existing experimental constraints and there is a chance to test the scenario at the LHC.

As in the standard Peccei-Quinn scenario the effective axion coupling $(a/f_a + \theta_{\rm eff})$ relaxes to zero through instanton effects, solving the strong CP problem dynamically. The axion mass is generated in a standard manner [9]

$$m_a = \frac{f_\pi m_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} = 0.6 \text{ eV} \frac{10^7 \text{ GeV}}{f_a} \,, \tag{23}$$

and no strictly massless scalars remain in the spectrum.

Let us discuss consequences of the remaining interactions in the 5D CS term (6) that consists of terms quadratic and quartic in the non-zero KK modes. We will focus (for obvious phenomenological reasons) on the quadratic terms shown explicitly in (18). Of course, there are other terms involving the heavy fields generated by the kinetic part of the action (3), those have have been considered previously in the literature, see e.g. [10] .

Because of its relatively large coupling, the very last term ($\propto m_n$) in (18), will produce the most noticeable effects at the LHC. Therefore let us consider the production of heavy gluons $G_\mu^{(n)}$ and vector bosons $X_\mu^{(n)}$ (with $n \geq 1$) at the LHC. At the partonic level the leading contributions are the following: $GG \to G^\star \to G^{(n)} X^{(n)}$ and $GG \to G^{(n)} X^{(n)}$. Since the SM fields do not carry $U(1)_X$ quantum numbers, the $X_\mu^{(n)}$ bosons are stable at the tree level; on the other hand, heavy gluons $G_\mu^{(n)}$ couple to SM quarks located on a brane. Therefore the experimental signature for the above reactions would be missing energy and momentum (carried away by the stable $X_\mu^{(n)}$) and two jets from the $G_\mu^{(n)}$ decays. Let us compare the amplitude strength for this process with the standard QCD two jet production amplitude. Adopting the estimate of the CS coupling from the NDA in (18) we find that the ratio of the $X_\mu^{(n)} G_\nu^{(n)} G_\alpha$ coupling to the SM triple gluon vertex is of the order of

$$\frac{g'}{g} \frac{\alpha_s}{3\pi} \, n \sim \frac{g'}{g} 10^{-2} \, n \tag{24}$$

Since $n \sim 1$ (otherwise KK modes are too heavy to be produced), it seems that it may be difficult to detect $G^{(n)} X^{(n)}$ over the two-jet QCD background. Nevertheless it should be noticed, that the huge amount ($\sim$ TeV) of missing energy (carried away by the stable and heavy $X_\mu^{(n)}$) may enhance the signal relative to the QCD background very efficiently, and that the large gluon luminosity of the LHC could be sufficient to provide enough events to test the scenario. Though these expectations are supported by the results for similar processes at the Tevatron [11], a dedicated Monte Carlo study would be needed to resolve this issue definitively; this, however, lies beyond the scope of this note.

Other possible signature of the axion being the 4th component of 5D gauge field could be the heavy gluon production process through a virtual axion exchange: $GG \to a^\star \to G^{(n)} G^{(n)}$ for $n \geq 1$. The amplitude for this process is generated by the first two terms in (18). It is straightforward to find that the order of magnitude for the amplitude normalized to two gluon (GG) production is the following:

$$\frac{\alpha'}{9\pi^2} \alpha_s \, n^2 \sim 10^{-3} \alpha' \, n^2 \,, \tag{25}$$

where $\alpha' \equiv g'^2/(4\pi)$. If $\alpha' \sim \alpha_s$ then for small n the amplitude is suppressed by the factor 10^{-4}. Since both $G^{(n)} G^{(n)}$ and GG states decay roughly the same way (the signature is $n \geq 4$ jets in the final state), it would be a real challenge to see the axion exchange over the standard QCD background [7].

Let us assume that the axion mass m_a (or equivalently the decay constant f_a) is known. Then the definite test of the model discussed here would be a verification of the gauge-axion unification that is caused by the fact that the axion is a component of the 5D gauge field X_M. The important consequence of the unification is that the total cross section for $G^{(n)} X^{(n)}$ production is predicted including the normalization. Therefore the measurement of $\sigma_{\rm tot}(G^{(n)} X^{(n)})$ shall provide the definite experimental test of the model.

Concluding the review of various possible experimental tests of gauge-axion unification discussed here, one can say that, because of a hudge missing energy ($\sim$ TeV), the process $GG \to G^{(n)} X^{(n)}$ provides the cleanest signature, that makes the observation of the signal plausible.

For the model being considered here the axion decay constant f_a is determined by the geometrical scale R^{-1} (if the NDA arguments are applied), therefore experimental limits on f_a constrain the size of the compact dimension dimension. However, it should be emphasized that most of these constraints rely on effects produced by the coupling

[7] Note also that the amplitude receives contributions from the other terms in the action.

of the axion to two photons, and this coupling is *absent* in our model (to leading order). (For a review of experimental constraints see [6].) Nevertheless there exists a bound that should be obeyed also by our photofobic axion; this is the so called "misalignment" lower axion mass limit that originates from the requirement that the contribution to the cosmic critical density from the relaxation of the axion field ($\theta_{\rm eff} \to 0$) does not overclose the universe. The resulting constraint [6], $m_a > 10^{-6}$ eV, leads to $R^{-1} \lesssim 10^{13}$GeV, having used (22-23) and taken $g^{'} = \mathcal{O}(1)$. Note that the NDA estimate of the CS coupling was crucial to derive the limit on R.

IV. CONCLUSIONS

We shown that an extension of naive dimensional analysis to 5D gauge theories naturally allows relatively large coefficients in front of Chern-Simons (CS) terms. The strong CP problem was discussed within a simple scenario containing a new $U(1)_X$ gauge field and the $SU(3)_{\rm color}$ gauge fields propagating in the bulk, and interacting through a a mixed CS term. Adopting appropriate boundary conditions, the CS term was shown to be gauge invariant (without any need for brane matter). The zero mode of the extra component of the new Abelian gauge field was seen to play a role of the axion (gauge-axion unification), which in the standard manner receives the instanton-induced potential, so that the strong CP problem (localized on the branes) disappears while the axion receives a mass. In the effective low-energy regime, the axion couples only to gluons, therefore most of the limits on the axion decay constant do not apply in the context of this model. It was shown that the most promising test of the gauge-axion unification is the process of $G^{(n)}X^{(n)}$ production: $GG \to G^{(n)}X^{(n)}$. The hudge missing energy ($\sim$ TeV) carried away by the stable and heavy $X_\mu^{(n)}$ is believed to provide a sufficiently clean signature of the final state.

APPENDIX

In this appendix we provide, for completeness, a summary of the application of Naive Dimensional Analysis (NDA) to higher-dimensional models. The NDA allows to determine the scale Λ at which the theory becomes strongly interacting. For that purpose let us compare two graphs with the same number of external legs, one of which has an additional gauge-boson propagator. This second graph will be suppressed with respect to the first by the factor

$$\Lambda^\delta g^2 l_{4+\delta}^{-1}; \quad l_D = (4\pi)^{D/2}\, \Gamma(D/2)\,, \tag{26}$$

where g denotes the gauge coupling constant, and l_D is the geometric loop factor obtained form integrating over momentum directions (note that in $D = 4+\delta$ dimensions g has a mass dimension of $-\delta/2$). For a strongly interacting theory we impose the NDA requirement that the loop corrections be of the same order as the lowest-order value; this requires

$$\Lambda \sim \left(l_{4+\delta} g^{-2}\right)^{1/\delta}\ . \tag{27}$$

The same NDA requirement allows an estimate of the coefficients in front of effective operators. For this we consider a generic vertex of the form

$$\mathcal{V} = \lambda\, \Lambda^D (2\pi)^D \delta^D (\sum p_i) \left(\frac{g\,\psi}{\Lambda_\psi^{3/2}}\right)^f \left(\frac{p}{\Lambda}\right)^d \left(\frac{g\,A_M}{\Lambda}\right)^b \left(\frac{g\,\phi}{\Lambda_\phi}\right)^{b'}\ ; \tag{28}$$

where scale appropriate for the vector fields and derivatives (they enter together through the covariant derivative) was chosen to be Λ, while the coefficient λ, the fermionic scale (Λ_ψ) and the scalar scale (Λ_ϕ) are to be determined. The requirement to reproduce the starting operator by radiative corrections determines the maximal value of λ and minimal scales Λ_ψ, Λ_ϕ that are allowed by perturbativity

$$\lambda = l_{4+\delta}^{-1} \qquad \text{and} \qquad \Lambda_\psi = \Lambda_\phi = \Lambda. \tag{29}$$

Let us now restrict ourselves to 5d theories, $\delta = 1$, and define the "index" of a vertex by

$$s = d_c + b' + \frac{3}{2} f - 4; \quad d_c = d + b\,. \tag{30}$$

where d_c is the number of covariant derivatives present in the vertex $\mathcal{V}$. If an L-loop graph contains V_n vertices with indices s_n, then the vertex corresponding to this graph has an index

$$s = L + \sum_n V_n s_n. \tag{31}$$

In terms of s the coefficient of a given operator is (see also [12])

$$\left(\frac{1}{24\pi^3}\right)^s \times (\text{the powers of } g \text{ needed to get a dimension 5 object})\,; \tag{32}$$

and $\Lambda = 24\pi^3/g^2$.

If the indices of all vertices are non-negative, then it follows from (31) that $s \geq s_n$ for all n. This implies that if $\mathcal{V}$ has index s, then only operators with indices $\leq s$ can renormalize the coefficient of $\mathcal{V}$ and we can then define a hierarchy according to the value of s in the sense that we can consistently assume that operators with higher indices are generated only by higher orders in the loop expansion . This would be spoiled if the theory has vertices with negative indices, (as an addition of an internal line attached by vertices with $s_n < 0$ decreases s, so an extra loop leads to less suppressed operator) which corresponds to the case $d_c = f = 0,\ b' = 3$, according to the definition (30). In order to define a hierarchy one should accordingly require that all terms cubic in the scalar fields be absent [8] due to an additional symmetry such as a discrete $\mathbb{Z}_2$ under which the ϕ are odd, by gauge invariance, (as in the SM) or just by an absence of scalar fields (as in this note where we are considering only vector bosons in 5D therefore the cubic scalar interactions cannot be constructed and the hierarchy of operators is given just by (32) without any other constraints). Fermion fields are assumed to transform appropriately under this symmetry, so as to allow all desirable scalar-fermion couplings.

In order to include consistently possible brane terms in the hierarchy we note that this type of interactions are naturally generated by the bulk terms in a compactified space at the one loop level [13]. It is then natural add 1 to s whenever a localizing factor of the form $\delta(y - y_o)$ is present. In addition the geometric suppression factor for these terms equals $l_4 = 16\pi^2$ that replaces $l_5 = 24\pi^3$ present in (32); see also [14].

Acknowledgments

This work is supported in part by the Ministry of Science and Higher Education (Poland) in years 2006-8 as research project N202 176 31/3844, by EU Marie Curie Research Training Network HEPTOOLS, under contract MRTN-CT-2006-035505 and by the U.S. Department of Energy grant No. DE-FG03-94ER40837. B.G. acknowledges the support of the European Community under MTKD-CT-2005-029466 Project, he also thanks Jacek Pawelczyk and Kin-ya Oda for their interest at the beginning of this project, and Zygmunt Lalak for being a patient witness of his struggle while this work was emerging.

[1] S. M. Barr, G. Segre and H. A. Weldon, Phys. Rev. D **20**, 2494 (1979).
[2] B. Grzadkowski and J. Wudka, Phys. Rev. Lett. **93**, 211603 (2004) [arXiv:hep-ph/0401232]; Phys. Rev. D **72**, 125012 (2005) [arXiv:hep-ph/0501238]; Acta Phys. Polon. B **36**, 3523 (2005) [arXiv:hep-ph/0511139].
[3] G. Aldazabal, L. E. Ibanez and A. M. Uranga, JHEP **0403**, 065 (2004) [arXiv:hep-ph/0205250]; K. I. Izawa, T. Watari and T. Yanagida, Phys. Lett. B **534**, 93 (2002) [arXiv:hep-ph/0202171]; K. W. Choi, Phys. Rev. Lett. **92**, 101602 (2004) [arXiv:hep-ph/0308024]; A. Fukunaga and K. I. Izawa, Phys. Lett. B **562**, 251 (2003) [arXiv:hep-ph/0301273]; R. Harnik, G. Perez, M. D. Schwartz and Y. Shirman, JHEP **0503**, 068 (2005) [arXiv:hep-ph/0411132].
[4] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).
[5] K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979); Phys. Rev. D **21**, 2848 (1980) [Erratum-ibid. D **22**, 1499 (1980)].
[6] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
[7] C. T. Hill, Phys. Rev. D **73**, 085001 (2006) [arXiv:hep-th/0601154].
[8] Y. Hosotani, S. Noda, Y. Sakamura and S. Shimasaki, Phys. Rev. D **73**, 096006 (2006) [arXiv:hep-ph/0601241].
[9] H. Y. Cheng, Phys. Rept. **158**, 1 (1988).
[10] D. A. Dicus, C. D. McMullen and S. Nandi, Phys. Rev. D **65**, 076007 (2002) [arXiv:hep-ph/0012259]; A. Muck, A. Pilaftsis and R. Ruckl, Phys. Rev. D **65**, 085037 (2002) [arXiv:hep-ph/0110391].
[11] T. Han, D. L. Rainwater and D. Zeppenfeld, Phys. Lett. B **463**, 93 (1999) [arXiv:hep-ph/9905423]; A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97**, 171802 (2006) [arXiv:hep-ex/0605101]; [D0 Collaboration] D0 CONF 4400 v1.4 http://www-d0.fnal.gov/Run2Physics/WWW/results/prelim/NP/N06/N06.pdf
[12] H. Georgi, Phys. Lett. B **298**, 187 (1993) [arXiv:hep-ph/9207278].

[8] This statement holds within NDA, of course, if the coefficients of super-renormalizable operators are tuned to be small, then their effects are suppressed so that the problem of consistency disappears.

[13] H. Georgi, A. K. Grant and G. Hailu, Phys. Lett. B **506**, 207 (2001) [arXiv:hep-ph/0012379].
[14] Z. Chacko, M. A. Luty and E. Ponton, JHEP **0007**, 036 (2000) [arXiv:hep-ph/9909248].